OPERA neutrino anomaly is a result of not interpreting energy uncertainty.

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Abstract

In this paper we bring out a remarkable consistency of theory of Relativity in explaining the anomalous excess of speed of neutrinos observed in the recent baseline experiment of OPERA. The OPERA experiment is performed by shooting neutrinos produced from protons at SPS, CERN to the laboratory at Gran-Sasso where OPERA has placed its neutrino brick detectors. We believe this result was misinterpreted to claim a super-luminal neutrino. The energy uncertainties inherently present in the OPERA neutrino measurement have not been reported on the claims of speed excess. The basics of Quantum Mechanics on the kinematic aspects of these neutrinos have been pointed out in this paper. We make a minimal review of this negligence of uncertainties which is sufficient to see where OPERA has lacked a cautious sight in claiming super-luminal neutrinos. We perform a rigorous check of Quantum Mechanics uncertainty principle of Energy-Time to make our claim of lack of any evidence of super-luminal neutrino.

I. INTRODUCTION

In this paper we provide a stringent condition on the minimum uncertainty on energy one deals with, on any particle with energy E momentum p and rest mass m. We find a relation between the uncertainty on speed and time following directly from the uncertainty of energy and time. Our relation is general and expressed in terms of the Compton wavelength of any particle, in particular the neutrino from OPERA experiment [Ref: 6]. This is a very accurate form of **speed-time** uncertainty relationship from **energy-time** uncertainty relationship. In our calculations we have made careful attempts to be consistent with the units of **speed-of-light**. Our result is valid for ultra-relativistic conditions of OPERA as much as it is valid for any speed of the particle, down to the lowest β one can theorize. All we do for OPERA situation is let our $\beta \to 1$. We do not use ultra-relativistic conditions except when evaluating constants in the case of OPERA neutrinos. Our expressions are valid for a relativistic treatment of general nature.

We mention that any experiment and theory of nature in Physics to be consistent with what we know must confirm to the famous uncertainty principle of Heisenberg, where applicable. From the logical edifice of Relativity theory and it's most popular concepts follows that there are 3 uncertainty principles, but in terms of equivalence, only 1 uncertainty principle is chosen as per the necessity of the physical problem at hand. Here we chose the explicit form of Energy-Time uncertainty relationship, because a baseline speed measurement rests on such a situation.

The OPERA experiment measures it's neutrino speed by claiming a very precise timing aided by the GPS satellite system for such measurements. This entails them a millimeter level accuracy in distance and a ns level accuracy in time at-least as per the specification of their GPS receivers. Since we have done much prior analysis that shows that GPS satellites in their circular orbits are very very well understood as per special and general theory of relativity, we do not ascertain any source of inaccuracy here. We mention that gravity of earth size objects is $\{S_r = 2.GM_e\}$ in itself a millimeter level accuracy. The exact value depends on the exact parameters of the problem and the separation from the gravity-source. The $25\,ppm$ **speed-of-light** excess of OPERA experiment in terms of absolute speed is a $\sim 7.5\,km/s$ excess. Such a large fallout in the speed-of-light is an unexpectedly large fallout with respect to the theory of Relativity.

The conclusion we draw is one has a millimeter level accuracy on GPS distance and nanosecond level accuracy on GPS time. Hence one must see a millimeter level speed excess as in other cases of theory of **Relativity Paradigm** if interpreted correctly with the recognition that the further complicacies in OPERA situation comes for two reasons. i. We are dealing with elementary particles whose masses are the smallest we know in the physical world ii. These particles have speeds that are immensely relativistic. For these two reasons one does not see a minimum in the millimeter range. In-fact the reason-i is dominant as masses can vary over a wider scale. The relativistic factor; reason-ii, does not vary as much. e.g. the OPERA neutrino and any electron moving at about the same speed, reason-ii is the same factor. But for these two cases for neutrino the minimum uncertainty we find is at $2.09 - meters/seconds$ where as for electrons this will be $(0.511/2) \times 10^6$ times lesser than what it is for neutrinos. This is for a $10 - ns$ GPS aided time precision {and any type of time precision in general}. The electrons moving at about **speed-of-light** will be uncertain of their speed at-least by;

$$\left(\frac{4.18}{0.511}\right)\times 10^{-6}\, m/s = \mathbf{8.18}\,\mu\mathbf{m/s};\ \mathbf{electron's-minimum}.$$

We mention in advance that in this paper we determine for a $2\,eV$ neutrino a minimum of $\sim 2.09\,m/s$ uncertainty in speed either below or above speed of light.

$$\mathbf{2.09\,m/s};\ \mathbf{neutrino's-minimum}.$$

A reinterpretation or rather a correct analysis of OPERA paper would suggest that the millimeter level distances {of the GPS} were blown up in the actual data-analysis of the OPERA measurement since distances and energies are correlated in theory of Relativity by the well established energy-momentum relationship. This relationship assumes further degrees of complicacy in particle physics experiments when kinematic relations of various energy channels and detector responses are added. So we need to factor in all sources of energy uncertainties to see why OPERA seeing an anomaly of sorts is quite explainable by basic Physics. OPERA sets their neutrino masses to a nominal value of $2\,eV$ which means a 0 uncertainty on the mass but the total Energy/Momentum uncertainties do not vanish that way which increases the mass error again. We refer to a more general case of kinematic errors on neutrino mass. This resembles more to the method of MINOS experiment on neutrino speed[Ref: 9]. MINOS assigns their neutrinos a mass from the procedure of reconstruction in the detector itself. Our treatment is a general form for any sophisticated analysis in any kind of particle physics experiment or even a particle reaction out side of accelerators or detector.

II. RELATIVISTIC KINEMATICS AND QUANTUM MECHANICS

A. Uncertainty Relation of speed-time from energy-time.

The **"energy, mass, momentum"** equation usually called the **energy-momentum** relation [Ref: 2] is expressed in **speed-of-light** = 1 units as: $E^2 = m^2 + p^2$, so

$$\mathbf{E} = (\mathbf{m^2} + \mathbf{p^2})^{\mathbf{1/2}} \tag{II.1}$$

where m is the rest mass of the neutrino or any relativistic particle. We note that m can itself be a nominal value as used by OPERA or a further kinematic sequence as used by MINOS. This is actually the reason we suspect why MINOS does not see the anomalous effect with a higher significance as the uncertainties if present automatically take care of the validity of the uncertainty minimums. For a stronger claim one needs to factor in all the kinematic contribution of energy uncertainty on m and it follows the same analysis path as presented in this paper. We have given a general form of this in this report. One needs the exact kinematic channels so as to iterate correctly in the relativistic equations inherently present in eqn (II.1). The errors associated with energy from other sources can be placed by hand in our derived result later, if one knows such with precision. In general any result on speed is dominated by errors of distance/speed/energy as these are equivalents, given a fixed precision on time. MINOS has it's kinematic neutrino mass errors included in it's analysis, so some of the errors might be canceling each other although they do not have a statistically significant result. We do not know if MINOS also suffers the same errors as neglected by OPERA or not.

We differentiate the above, eqn (II.1) to see the relation between any shift or error in the above equation. That is the errors will be related in the differentials, given by:

$$\Delta E = \{1/2\} \times (m^2 + p^2)^{-1/2} \times 2 \times \{m\Delta m + p\Delta p\}$$

This analysis does not differentiate between the forward, backward or central differentials, so you can use any; delta = Δ = forward, anadelta = ∇ = backward and *delta* = δ = central difference. Now let us apply the Heisenberg's energy-time uncertainty relationship, $\Delta E.\Delta t \geq \hbar$;

$$\Delta E.\Delta t = (m^2 + p^2)^{-1/2}(m\Delta m + p\Delta p).\Delta t \geq \hbar,$$

so,

$$(m^2 + p^2)^{-1/2} m.\Delta m.\Delta t + (m^2 + p^2)^{-1/2} p.\Delta p.\Delta t \geq \hbar.$$

So we have now,

$$(1 + \gamma^2\beta^2)^{-1/2}\Delta m.\Delta t + (1 + \gamma^2\beta^2)^{-1/2}\gamma\beta.\Delta p.\Delta t \geq \hbar$$

in the left we used $p = m\gamma\beta$, so naturally

$$\Delta p = (\Delta m)\gamma\beta + m.\Delta(\gamma\beta). \tag{II.2}$$

Using this, eqn(II.2);

$$\frac{[(\mathbf{1} + \gamma^{\mathbf{2}}\beta^{\mathbf{2}}).\mathbf{\Delta m.\Delta t} + \mathbf{m.c_b}.\gamma\beta.\mathbf{\Delta}\beta.\mathbf{\Delta t}]}{(\mathbf{1} + \gamma^{\mathbf{2}}\beta^{\mathbf{2}})^{\mathbf{1/2}}} \geq \hbar \tag{II.3}$$

where

$$c_b = \left(\frac{d(\gamma\beta)}{d\beta}\right)_{\beta\to 1} = \left(\frac{\Delta(\gamma\beta)}{\Delta\beta}\right)_{\beta\to 1}.$$

We also define $d_b = (\gamma\beta)_{\beta\to 1}$.

These definitions do not take away the generality as long as they have not been evaluated. So we can change our $\beta \to 1$ limit and re-evaluate the constants. Let us take the $\Delta m.\Delta t \sim \hbar$ limit which says any uncertainty on m is a minimum in that limit, so we have

$$(1+\gamma^2\beta^2)\hbar + m.c_b.\gamma\beta.\Delta\beta.\Delta t \;\geq\; \hbar\sqrt{1+\gamma^2\beta^2} \tag{II.4}$$

Note that setting $\Delta m.\Delta t \sim \hbar$ still does not make the minimum $\Delta E.\Delta t \sim \hbar$, in other words eqn(II.4) is not an equality yet, which is only consistent.

$$\mathbf{m.c_b}.\gamma\beta.\boldsymbol{\Delta}\beta.\boldsymbol{\Delta}\mathbf{t} \;\geq\; \hbar\left(\sqrt{\mathbf{1}+\gamma^{\mathbf{2}}\beta^{\mathbf{2}}} - \mathbf{1} - \gamma^{\mathbf{2}}\beta^{\mathbf{2}}\right) \tag{II.5}$$

$$\frac{m.c_b.\gamma\beta}{\sqrt{1+\gamma^2\beta^2}-(1+\gamma^2\beta^2)}.\Delta\beta.\Delta t \geq \hbar$$

$$\frac{m.c_b.\gamma\beta}{\sqrt{1+d_b^2}-(1+d_b^2)}.\Delta\beta.\Delta t \geq \hbar$$

$$\Delta\beta.\Delta t \geq \frac{\hbar}{m.c_b.d_b}.\left(\sqrt{1+\gamma^2\beta^2}-1-\gamma^2\beta^2\right)$$

$$\Delta\beta.\Delta t \geq \frac{\lambda_c}{c_b.d_b}.\left(\sqrt{1+\gamma^2\beta^2}-1-\gamma^2\beta^2\right)$$

$$\Delta\beta.\Delta t \geq \lambda_c.\left(\frac{\sqrt{1+d_b^2}}{c_b.d_b} - \frac{1+d_b^2}{c_b.d_b}\right)$$

We give a general description of this in the note where we do not set $\Delta m.\Delta t \sim \hbar$. Also it is worthwhile to mention here that $\Delta\beta$ in the above equations is $\Delta\beta_c =$ causality violation uncertainty which is necessarily $-ve$. We can intuit this if we say $\pm abs(\Delta\beta_c) = \Delta\beta$ where $\Delta\beta$ is actual uncertainty on speed which can be blown up by errors from a variety of sources.

B. The OPERA neutrino speed excess

All the above 5 equations followed by eqn (II.5) are the general form of **speed-time** uncertainty relation. Also we have lost the generality of uncertainty on mass m at this point. The generality can be reverted by not employing the uncertainty relation $\Delta m.\Delta t \sim \hbar$. These 5 equations are chosen to a given accuracy and in a given relativistic limit. We have employed the summation of binomial [Ref: 1] coefficients to determine c_b, d_b hence the subscript b. Later we will give the details how we determined these constants for OPERA neutrino situation. They are for OPERA neutrino, given by $c_b = 15.006$ and $d_b = 3.942$ which reminds us that β and γ are ultra-relativistic.

A note of caution; these constants have been adjusted for a **momentum-order** calculation. These may change therefore for **mass-order** and **energy-order** calculations. For **mass-order** they are found to be $\sim 10^{-8}$. λ_c is reduced **Compton Wavelength**. We evaluate the above equations in terms of known values and we have

$$\boldsymbol{\Delta}\beta_{\mathbf{c}}.\boldsymbol{\Delta}\mathbf{t} \geq -\mathbf{0.211}.(\lambda_\nu)$$

This is not only valid for neutrinos but any particle that is moving at or near $speed = \beta = 1 \equiv 3.0\times10^8 m/s$. We derived c_b, d_b to the order β^{10} at the limit $\beta \to 1$. We will attempt a more rigorous review of the evaluation of these constants in a later communication. But for now after several iterations and the fact that summing of binomial coefficients must in the end give only a value that does not change widely is enough to make a claim that our result is correct. The $-ve$ sign comes because $\Delta\beta$ is a causality violation limit. In this limit the particle is going below $\beta = 1$. It's an uncertainty. One can also say the minimum uncertainty $\Delta\beta$ is restricted by the Compton wavelength. With that in mind

$$\boldsymbol{\Delta}\beta.\boldsymbol{\Delta}\mathbf{t} \geq \mathbf{0.211}.(\lambda_\nu) \tag{II.6}$$

or,

$$\Delta\beta \geq \frac{0.211\times6.6\times10^{-7}\times eV.ns}{2\times10\times eV.ns}$$

or $\Delta\beta \geq 0.696\times10^{-8}$, for $c = 3.0\times10^8\, m/s$ this is $\Delta\mathrm{v} \geq 2.09\, m/s$. One concludes OPERA must see a minimum of $2.09\, m/s$ at a precision of $10\, ns$. {for $1\, ns$ one just multiplies by 10, for speed, energy, momentum} We see that $\{\boldsymbol{\Delta}\beta,\, \boldsymbol{\Delta}\mathbf{p}\} \approx \mathbf{6.6}\times\mathbf{10^{-4}eV}$, for a $10\, ns$ precision to see a $2.09\, km/s$ uncertainty in the speed of neutrino. This uncertainty is for **momentum-order**, for **mass-order** one divides by $c = 3\times10^8$ and for **energy-order** one multiplies the **momentum-order** by $c = 3\times10^8$. So we have for $2.09\, km/s$ uncertainty, $\boldsymbol{\Delta}\mathbf{E} \sim \mathbf{19.8\, KeV}$, $\Delta p \sim 660\, ppm\, of\, 1\, eV$, $\Delta m \sim 0.22\times10^{-12}\, eV$.

One sees therefore that if OPERA incurs an uncertainty on it's energy, mass or momentum measurement of the neutrino a very small value given by $\boldsymbol{\Delta}\mathbf{E} \sim \mathbf{19.8\, KeV}$, $\Delta p \sim 660\, ppm\, of\, 1eV$, $\Delta m \sim 0.22\times10^{-12}\, eV$ the super-luminal claim of $7.5\, km/s$ can still be valid. One needs to tighten the uncertainties on E, p, m a little more, as we need $7.5\, km/s$ not the lesser $2.09\, km/s$ as we did. So, one needs to blow up the constraint by multiplying a factor $\frac{7.5}{2.09} = 3.589$, in other words OPERA's super-luminal claims vanish at $\boldsymbol{\Delta}\mathbf{E} \sim \mathbf{71.06\, KeV}$, $\Delta p \sim 2.37\, meV$ or $\Delta m \sim 0.79\times10^{-12}\, eV$.

C. Errors on energy from recent experiments

The recent particle physics experiments such as belle at KEK, Japan have an uncertainty of $\sim 500\, KeV$ on their center of mass energy of $\sim 5.4\,GeV$. This is a result from $2010-2011$ [Ref: 7]. The belle uncertainty at same center of mass energy was $\sim 800\,KeV$, in a highly cited paper from 2003 [Ref: 8]. This suggests that while the techniques of reconstruction have improved it has not come down to a value of $100\,KeV$. OPERA could survive $10\,KeV$ but not $100\,KeV$. This raises a very pertinent question on what OPERA could achieve in terms of their energy uncertainty since that is also a particle physics experiment. Without the actual statement of uncertainty on their energy it is not at all safe to make a super-luminal claim.

In other words Quantum Mechanics does not exclude super-luminal neutrinos, it imposes an extremely harsh condition on the precision of energy. One needs in the entirety of one's analysis to be able to see if there is any super-luminal excess or not. Following our discussion from **subsection-B** (II B) above, in-fact, OPERA claims a $1.23\,km/s$ uncertainty, this is possible only when they have slightly worse uncertainty on their energy of $17\,GeV$ than $10\,KeV$ as you can see above {actually $11.65\,KeV \approx 1.23\,km/s$}. At $100\,KeV$ their super-luminal claim vanishes, actually as we just showed it vanishes at $71\,KeV$ on their $17\,GeV$ energy which is a $4.18\,ppm$ uncertainty on energy. $11.65\,KeV \approx 1.23\,km/s$ is a $0.685\,ppm$ uncertainty on their energy.

III. NOTES

1. We need a subtle point of relativity in our calculations, to be consistent with the overall results of our analysis. Here is the actual equivalence: we realize that they are present in various important equations established by theory of relativity and used frequently in relativistic applications/investigations. SO the total set of equivalence is E, p, v, m, d, t. The 3 Quantum Mechanics uncertainty principles are all constituted from among these variables, hence they are all equivalently only 1 equation but appear in different forms if we start from one and derive another. We notice that d, t are also equivalenced like the famous E, m. The other important property to notice is d and t are chosen for the parametrization of kinematics and all the other variables here are paired for canonical commutation with respect to d and t.

2. When The set of equivalent parameters $\{E,\ p,\ \mathrm{v},\ m,\ d,\ t\}$ is commuted with either d or t, one at a time and excluding them from the main set of parameters the commutation produces the uncertainty in the order of Planck's constant $\hbar$, which is the reduced Planck's constant. The uncertainty of commutation bears a very simple inequation for 3 specific cases when these are called the uncertainty relationship or (in)equations in Quantum Mechanics. But in conjugation with relativity as we mentioned already one can start with only one uncertainty relation and observe the other two by employing the "classical" or relativistic definitions of the other parameters as per suitability of the problem. Then from a simple relationship of commutation-uncertainty one arrives at more complicated relationships which in specific cases and in the limits of minimum uncertainty returns to the simpler form again. We also note that the simpler form of uncertainty can be rendered more complicated in the realm of particle physics as more than one kinematic contributions appear and as detector responses are factored into the uncertainty behavior.

3. The uncertainty of d and t go opposite to the uncertainty of the other variables $\{E,\ p,\ \mathrm{v},\ m\}$ to the order of either $\hbar$ or in consistency of units $\hbar/m_0$. $\hbar/m_0$ in speed-of-light units is called a **Reduced Compton Wavelength** of the particle represented by mass m_0. Additional **speed-of-light** unit consistency check is needed at several levels of the calculation. From an equivalence we do not always use an uncertainty equation in the same variable. They need to be equated in a correct dimensional analysis. This can easily be done by employing a speed-of-light unit. We were careful in this paper with the units and dimensions so as not to incur incorrect values. d, t define the speed v, hence we have also included the v in the set of equivalent variables.

4. How do we sum our binomial coefficients? We use a bound on summation of binomial coefficients given by Michael Lugo: [Ref: 4];

$$\mathbf{f\,(n,\,k) \leq \binom{n}{k}\,\frac{n-(k-1)}{n-(2k-1)}}.$$

   On MathOverFlow.Net {Sum of the first k binomial coefficients} Michael Lugo gives two bounds on summing the binomial coefficients, one for a fixed k which we use in this paper and one for $k = N/2 + \alpha\sqrt{N}$. Because of the method of summing the binomial coefficient and the exact order of β we will incur a very slight error on the constraint we provide on the uncertainty of E. This does not change the order of the energy uncertainty ΔE as binomial coefficients are fractions, we summed to a very high degree already.

5. Here we give the details of summing the binomial coefficients in the expansion of the Lorentz Factors in the limit of $\beta \to 1$. Here we refer to an analysis we have done in determining the binomial expansion of the Lorentz Factors and their power func-

tions [Ref: 5] .

$$f(\beta) = \beta(1-\beta^2)^{-1/2} = \beta \left\{ \sum_{k=0}^{\infty} \binom{-\frac{1}{2}}{k} (-\beta^2)^k \right\} \tag{III.1}$$

We want $f(1)$;

$$f(1) = \sum_{k=0}^{\infty} \binom{-\frac{1}{2}}{k} (-1)^k. \tag{III.2}$$

According to a bound given in **MathOverFlow.Net** as refered in Note-(4), [Ref: 4]

$$\mathbf{f(1)} = \sum_{\mathbf{k=0}}^{\infty} \binom{-\frac{\mathbf{1}}{\mathbf{2}}}{\mathbf{k}} (-\mathbf{1})^{\mathbf{k}} \leq \binom{-\frac{1}{2}}{k} \frac{-\frac{1}{2}-(k-1)}{-\frac{1}{2}-(2k-1)}\{(-1)^k\} \tag{III.3}$$

Let us take $k=10$, i.e. $\sim \beta^{20}$.

$$f(1) = \binom{-\frac{1}{2}}{10} \frac{-\frac{1}{2}-(10-1)}{-\frac{1}{2}-(20-1)}\{(-1)^{10}\} = \frac{9.5}{19.5}\binom{-\frac{1}{2}}{10}$$

$$f(1) = \frac{9.5}{19.5}\frac{(-\frac{1}{2})!}{(-\frac{1}{2}-10)!(10)!} = \frac{9.5}{19.5}\frac{(-0.5)}{(-10.5)\times 3628800}$$

$$f(1) = -\frac{95\times 5\times 10^{-2}\times 2.76\times 10^{-5}}{195\times 105} = 1.314\times 10^{-8}$$

As we had noted earlier a **mass-order** value is in the $\mathbb{O}(\mathbf{10^{-8}})$ and we need to multiply for consistency of **speed-of-light,** $c = 3.0\times 10^8\, m/s$ everywhere there is a **m-term**, which is the case for $f(1) = \sum_{\beta\to 1}(\gamma\beta)$. So we have

$$\mathbf{f(1)} = \sum_{\beta\to\mathbf{1}}(\gamma\beta) = \mathbf{d_b} = \mathbf{3.942}. \tag{III.4}$$

Now we evaluate the constant

$$c_b = \left(\frac{d(\gamma\beta)}{d\beta}\right)_{\beta\to 1} = \left(\frac{\Delta(\gamma\beta)}{\Delta\beta}\right)_{\beta\to 1}.$$

We differentiate $\gamma\beta = \frac{\beta}{\sqrt{1-\beta^2}}$,

$$\frac{d}{dx}\left(\frac{F_1}{F_2}\right) = \frac{F_2.F_1^{'} - F_1.F_2^{'}}{F_2^2}$$

$$c_b = \frac{\sqrt{1-\beta^2}+2\beta^2(\sqrt{1-\beta^2})^{-1}}{1-\beta^2})^{3/2} = (1-\beta^2)^{-1/2} + 2\beta^2(1-\beta^2 \tag{III.5}$$

We define $c_b = g(\beta) + h(\beta)$ with

$$h(\beta) = 2\beta^2(1-\beta^2)^{3/2} \tag{III.6}$$

and $g(\beta) = f(\beta)$ if $\beta = 1$.

$$h(\beta) = 2\beta^2 \left\{ \sum_{k=0}^{\infty} \binom{-\frac{3}{2}}{k} (-\beta^2)^k \right\}. \tag{III.7}$$

$$h(1) = 2\left\{ \sum_{k=0}^{\infty} \binom{-\frac{3}{2}}{k} (-1)^k \right\}^{k=10} \leq 2\binom{-\frac{3}{2}}{10}\frac{-\frac{3}{2}-9}{\frac{3}{2}-19} \tag{III.8}$$

$$h(1) = 2\times\frac{10.5}{20.5}\frac{(-\frac{3}{2})!}{(-\frac{3}{2}-10)!(10)!} = \frac{9.5}{19.5}\frac{(-0.5)}{(-10.5)\times 3628800}$$

$$h(1) = \frac{105\times 2\times 15\times 10^{-2}\times 2.76\times 10^{-5}}{115\times 205} = 3.688\times 10^{-8}$$

$$g(1)+h(1) = \sum_{\beta\to 1}\frac{d}{d\beta}(\gamma\beta) = c_b = (3.688+1.314)\times 10^{-8}$$

We multiply here $c = 3.0\times 10^8\, m/s$ like earlier, this brings **mass-terms** and **momentum-terms** to the same order. We obtain

$$\mathbf{g(1)+h(1)} = \sum_{\beta\to\mathbf{1}}\frac{\mathbf{d}}{\mathbf{d}\beta}(\gamma\beta) = \mathbf{c_b} = \mathbf{15.006} \tag{III.9}$$

This concludes our method of evaluating followed by summing the binomial coefficients in the expansion of the Lorentz Factors and their power functions. As noted we have evaluated these in the **momentum-order**.

6. After we performed our calculations and see that OPERA experiment not citing the uncertainty on their energy as a reason of why they see this anomaly we also realized that one of our previous analysis would have led to the same conclusion if we were to correctly interpret that equation. This equation we are referring is used in the text of Weinberg [Ref: 3]. This can be considered an uncertainty on **proper-time**. This is not mentioned in terms of Compton wavelength of the participating particle or an uncertainty on proper-time. We change the ideas slightly and use it for an uncertainty on space and time;

$$(\mathbf{\Delta x})^{\mathbf{2}} - (\mathbf{\Delta t})^{\mathbf{2}} \leq \left(\frac{\hbar}{\mathbf{m_0}}\right)^{\mathbf{2}} \tag{III.10}$$

, note the sign of inequality here and the fact that

$$\left(\frac{\hbar}{m_0}\right)^2 = \lambda_c^2.$$

So,

$$(\Delta t)^2.\left\{\left(\frac{\mathrm{v}}{c}\right)^2-1\right\}\leq\left(\frac{\hbar}{m_0}\right)^2,(\beta+\Delta\beta_c)^2\leq 1+\frac{\lambda_c^2}{(\Delta t)^2} \tag{III.11}$$

where $\Delta\beta_c$ is the speed-excess or causality violation in terms of speed if $\beta\rightarrow 1$. Since $0\leq\beta\leq 1$ the last (in)equation means

$$\beta^2+(\Delta\beta_c)^2+2.\beta.\Delta\beta_c<1+\left(\frac{\lambda_c}{\Delta t}\right)^2+2.\frac{\lambda_c}{\Delta t} \tag{III.12}$$

as $\frac{\lambda_c}{\Delta t}$ is $+ve$. If $\beta\rightarrow 1$ this means

$$(1+\Delta\beta_c)^2<\left(1+\frac{\lambda_c}{\Delta t}\right)^2 \tag{III.13}$$

, or

$$\mathbf{\Delta}\beta_\mathbf{c}<\frac{\lambda_\mathbf{c}}{\mathbf{\Delta t}} \tag{III.14}$$

One can see that the last expression is "exactly" what we had obtained in our analysis at the top. In-fact $\Delta\beta_c$ is a causality violation excess at this limit of $\beta\rightarrow 1$, so we can change its sign and we have

$$\mathbf{\Delta}\beta>\frac{\lambda_\mathbf{c}}{\mathbf{\Delta t}} \tag{III.15}$$

By summing binomially and seeing the fact that the mass m can be really small such as that of **neutrino** we see that we have a factor 0.211 in front of λ_c. This is to be expected since for the equality above that we had removed will now be valid. So we approach the equality; binomially and asymptotically. The **Compton-wavelength-limit** corresponds to an inequality. The sources of errors such as detector response will **add-on** to the **Compton-Wavelength**. In-fact there never is an equality since it is asymptotic. That is, the factor 0.211 is only approximate although very very accurate. So, these lighter particles are capable of **off-setting** the causality violation excess by say a factor 0.211 but not completely make it zero. One can continue to sum the binomial coefficients to a larger and larger value and have a lighter and lighter mass but never completely breakaway from the photon speed. For electron which is much more massive than the neutrino, therefore, one would expect a factor which is <0.211 and one must see **speed-uncertainties** larger than any speed-excess above **speed-of-light**. Although

$$(\beta+\Delta\beta_c)^2<\left(1+\frac{\lambda_c}{\Delta t}\right)^2\Rightarrow\Delta\beta_c<\frac{\lambda_c}{\Delta t},\ \beta\rightarrow 1 \tag{III.16}$$

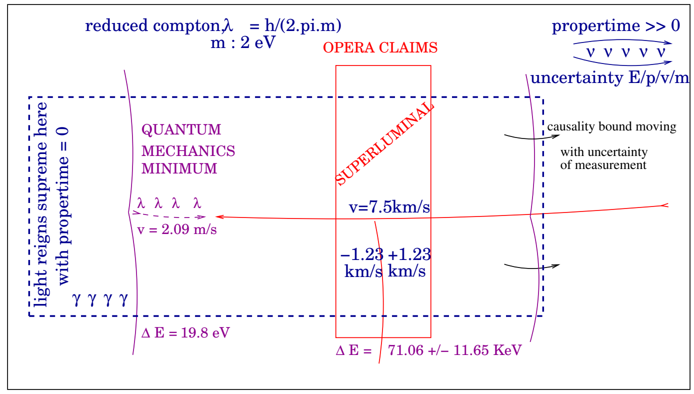


Figure III.1: Basic Quantum Mechanics: The super-luminal neutrinos need to have an energy precision better than energy uncertainty corresponding to reduced Compton wavelength of the neutrinos

(in the $\beta\rightarrow 1$ limit) this is also valid in contrary since $0\leq\beta\leq 1$. This is evident because if $\beta\ll 1$ one needs a "additive" correction factor to $\Delta\beta$ which then makes the

$$(\beta+\Delta\beta_c)^2<\left(1+\frac{\lambda_c}{\Delta t}\right)^2$$

valid/sufficient for inferring

$$\Delta\beta_c<\frac{\lambda_c}{\Delta t}$$

, this "additive" correction factor is $-(1-\beta)$, e.g. if $\beta=0.5$ then

$$\Delta\beta_c-0.5<\frac{\lambda_c}{\Delta t}$$

or

$$\Delta\beta_c<\frac{\lambda_c}{\Delta t}+0.5$$

and

$$\Delta\beta\geq\frac{\lambda_c}{\Delta t}+0.5$$

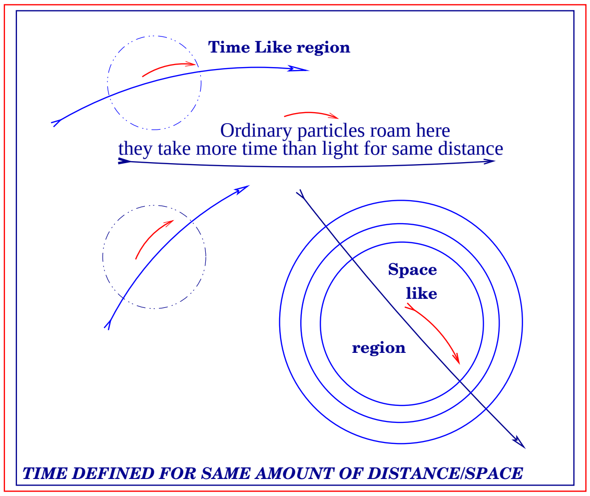
. Notice that $\Delta\beta$ and $\Delta\beta_c$ are different, $\Delta\beta_c$ refers to causality violation uncertainty hence $\Delta\beta_c$ is in the $\beta\rightarrow 1$ limit but $\Delta\beta$ refers to actual uncertainty on the speed. So in general if $0\leq\beta<1$;

$$\mathbf{\Delta}\beta\geq\frac{\lambda_\mathbf{c}}{\mathbf{\Delta t}}+(\mathbf{1}-\beta),\ \mathbf{0}\leq\beta<\mathbf{1}$$

IV. SUMMARY/CONCLUSION

The OPERA experiment is a particle physics experiment which has for 3 years accumulated a galore of neutrinos produced from the decay of protons. These protons

Figure III.2: **The situation in a super-luminal paradigm, theory of Relativity**



For the same amount of distance light produced a circle of time lesser in diameter to the length
that corresponds to other particles. {nothing takes lesser time than light}

On the shell:

$\delta\tau = 0$, $dt^2 - dx^2 = 0$

{for the distance and time to be equal only light has this property}.

$\delta\tau = 0$, $dt = dx$, $dx/dt = 1 = c$

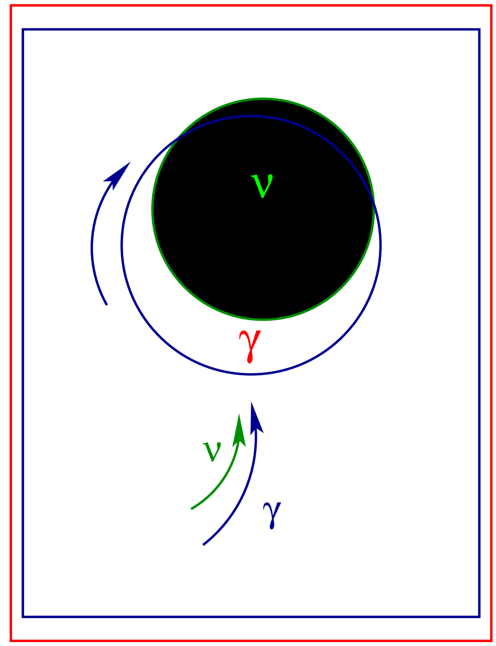
Inside circle/shell:

nothing, not even light moves here, it takes less time than light for same distance

NEUTRINOS?

If $\nu' s$ are super-luminal by default they will constitute a circle like this and photons
will be arrows outside the circle hovering in time−like regions defined by maximal ν speed,
but are there maximal ν speeds that can go above speed of light??

Figure III.3: The super-luminal paradigm, **what-if** OPERA is correct?



Not sitting on perfect circles/spheres??

perfect-sphere; $\delta\tau(x,t) = dt^2 - dx^2$

Is neutrino inconsistent with theory of Relativity,

$\delta\tau_\gamma = 0$**?**

$i.\, \delta\tau_\nu = ?, < 0?$

$ii.\, \delta\tau_\nu = g\,(E),\, \delta\tau_\gamma = f\,(E),\, g(E) < f\,(E)?$

were shoot from the laboratory at CERN, Geneva and the neutrinos produced from these passed through the earth-crust to another laboratory at Gran-Sasso, Italy which is located about 733 k.m.s, same as the coastline of Hong Kong or the distance of Oslo, Norway to Mo I Rana, Norway. The experiment recently claimed that as per its analysis aided by time precision of GPS satellites to about 10 nano-seconds they observe that their 15000 neutrinos would travel faster than light particles known as photons. Such neutrinos are called as super-luminal neutrinos as they can travel past lumina, that is light. This is not a very welcome observation since it puts into question Albert Einstein's firmly established Theory of Relativity and almost all the other branches of Modern Physics. In the last 3 months the world has seen about a 100 applications of the various knowledge segments of the physical world to explain or refute the findings. None of the findings were sufficient to disprove OPERA experiment's findings which had performed an analysis with an impressive statistical significance that is hardly ever achieved in such measurements.

We have deduced from basic energy-time uncertainty relation of Quantum Mechanics a very accurate form of speed-time uncertainty relation. This speed-time uncertainty relation imposes stringent conditions at what energy uncertainty OPERA can claim super-luminal neutrino and at what uncertainties it cannot. We follow basic conditions of special Theory of Relativity and Quantum Mechanics to claim that OPERA must have total energy uncertainties below ~ 71 kilo-electron-volts to consistently conclude that special theory of Relativity is inconsistent with their finding. We cite latest measurement uncertainties in some recent experiments of similar nature and similar energy scale to claim that such precision has yet not been achieved. The way forward for OPERA is to study and disclose their energy uncertainties. This will rest the matter of whether neutrino speed can go above the speed limit of nature imposed by special theory of Relativity of Albert Einstein. According to this theory only light particles known as photons travel at the speed limit of nature since they have no mass. Neutrinos are known to have some mass, although very very small compared e.g. to electrons, because they can change their form from one type to another. According to Theory of Relativity anything that has mass can never go above the photon's speed of $\sim (3 \times 10^5 = 3,00,000)\, km/s$.

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